RAN-2203000205023005

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T.Y.B.Sc. (Mathematics) (Sem. V) Examination November - 2023

MTH-505 - Graph Theory

[Total Marks: 50

સૂચના : / Instructions

(૧) **નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.**
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **T.Y.B.Sc. (Mathematics) (Sem. V)**

Name of the Subject :

☛ **MTH-505 - Graph Theory**

Subject Code No.: **2203000205023005**

Seat No.:

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the corresponding question.

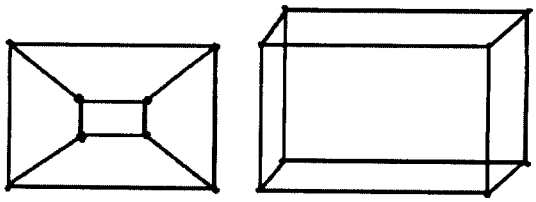
Q-1 Answer the following (any five): 10

1. Find the sum of degrees in a simple graph containing 5 vertices to the maximum.
2. Construct complete graphs with three and four vertices.
3. Is a polygon included in a circuit? Justify your answer.
4. Is every Euler graphs are Arbitrarily traceable ? Justify your answer.
5. State the seating arrangement of 11 members in which every member has different neighbors at a round table.
6. State sufficient condition for a simple graph G to have a Hamiltonian circuit.
7. Find the internal vertex in a binary tree having 15 vertices.
8. Is every path in a connected graph G being a tree? Justify your answer.

Q-2 Answer the following (any two) : 10

1. Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

2. Let n be the number of vertices of odd degree in a graph G then prove that n must be even.
3. State necessary condition for the graphs to be Isomorphic.
Verify that: The following two graphs are Isomorphic label the corresponding vertices and edges.



GRAPH G-1 GRAPH G-2

Q-3 Answer the following (any two): 10

1. Prove that a simple graph G with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges.

2. In a graph G, let p_1 and p_2 be two different paths between two given vertices. Prove that $p_1 \oplus p_2$ is a circuit or a set of circuits.

3. Let G be a graph containing $e_1, e_2, e_3, , , , , , , , , e_M$ edges then prove that it can be decomposed into $\left[\frac{2^M - 2}{2}\right]$ different ways into pair of sub graphs.

Q-4 Answer the following (any two): 10

1. Explain Konigsberg bridge problem.
2. Prove that : An Euler graph G is arbitrarily traceable from vertex v in G if and only if every circuit in G contains v.
3. (1) Is Hamiltonian path longer than any other path in a connected graph G? Justify your answer.
   (2) Prove that : Hamiltonian circuit can be constructed in a complete graph.

Q-5 Answer the following (any two): 10

1. Prove that: A connected graph G with n vertices and $(n-1)$ edges is a tree.
2. Let T be a binary tree with n vertices and p be the number of pendant vertex in it then prove that $p = \frac{(n+1)}{2}$.

3. Prove that: Every tree has either one or two centers.

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